HZPP-9905

July 25, 1999

An effective relaxation-time approach to collisionless quark-gluon plasma *

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July 29, 1999

Abstract

We present an effective relaxation-time theory to study the collisionless quark-gluon plasma. Applying this method we calculate the damping rate to be of order g^2T and find plasmon scattering is the damping mechanism. The damping for the transverse mode is stronger than the longitudinal one.

PACS number: 12.38.Mh

The collisional term in a transport equation is important for a many particles system. We see the transport properties are related to collisions, for example, the damping in plasma. Up to now, the collisional terms have not been resolved in quark-gluon plasma(QGP). We often study the case of collisionless plasma. Although it is well-known that the collective excitations

*Supported by the National Natural Science Foundation of China Grant No. 19805003

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make no contribution to transport processes for the ultra-relativistic plasma, we should believe the interactions of collective modes play vital role in the collisionless plasma. Here, we just study the damping due to the interactions.

We start from the following kinetic equations for QGP[1]

$$p^\mu D_\mu Q_\pm(\mathbf{p},x) \mp \frac{g}{2} p^\mu \partial_p^\nu \{F_{\mu\nu}(x), Q_\pm(\mathbf{p},x)\} = 0, \tag{1}$$

$$p^\mu \mathcal{D}_\mu G(\mathbf{p},x) - \frac{g}{2} p^\mu \partial_p^\nu \{\mathcal{F}_{\mu\nu}(x), G(\mathbf{p},x)\} = 0, \tag{2}$$

where the calligraphic letters represent the corresponding operators in adjoint representation of SU(3). Now, we separate the density fluctuations from the mean parts:$Q = \langle Q \rangle + Q^T, G = \langle G \rangle + G^T$, $\langle \ \rangle$ denotes the average over random phases[2]. Assuming the fluctuations to be weak, $p \sim gT, A_\mu \sim T, i\partial_\mu \sim gT$, we neglect the Abelian-like coupling of the fluctuations because the coupling is of higher order g than the non-Abelian one. Thus we write the equations for the fluctuations

$$p^\mu \partial_\mu Q_\pm^T \mp g p^\mu F_{\mu\nu} \partial_p^\nu Q_+^T = -igp^\mu \left([A_\mu, Q_+^T] - \langle [A_\mu, Q_+^T] \rangle \right), \tag{3}$$

$$p^\mu \partial_\mu G^T - g p^\mu \mathcal{F}_{\mu\nu} \partial_p^\nu G^T = -igp^\mu \left([\mathcal{A}_\mu, G^T] - \langle [\mathcal{A}_\mu, G^T] \rangle \right), \tag{4}$$

Effectively, we regard the nonlinear parts in the equations as 'collisional terms', which they represent the interactions of collective modes. We now define the effective collisional frequencies $\nu_q, \nu_{\bar{q}}$ and ν_g for quarks, antiquarks and gluons. In the relaxation-time approach, we have identities

$$p^\mu u_\mu \nu_q Q_\pm^T(k) = \int \frac{\mathrm{d}k_1 \mathrm{d}k_2}{(2\pi)^8} \mathrm{Im} \left(gp^\mu ([A_\mu(k_2), Q_\pm^T(k_1)] - \langle [A_\mu(k_2), Q_\pm^T(k_1)] \rangle) \right), \tag{5}$$

$$p^\mu u_\mu \nu_q G^T(k) = \int \frac{\mathrm{d}k_1 \mathrm{d}k_2}{(2\pi)^8} \mathrm{Im} \left(gp^\mu ([\mathcal{A}_\mu(k_2), G^T(k_1)] - \langle [\mathcal{A}_\mu(k_2), G^T(k_1)] \rangle) \right). \tag{6}$$

We can derive all orders of density fluctuations from the asymptotic method developed from weak-turbulent theory[3]. Here we only keep the first two orders. In the plasma rest frame, taking temporal gauge condition, we easily obtain

$$\nu_q \int \frac{\mathrm{d}\mathbf{v}}{4\pi} \langle Q_+^{(1)}(k)\mathbf{v}\cdot\mathbf{A}(k')\rangle = g \int \frac{\mathrm{d}k_1\mathrm{d}k_2}{(2\pi)^8}\delta(k-k_1-k_2)\mathrm{Im}\int \frac{\mathrm{d}\mathbf{v}}{4\pi}\langle[\mathbf{v}\cdot\mathbf{A}(k_2), Q_+^{(2)}(k_1)]\mathbf{v}\cdot\mathbf{A}\rangle, \quad (7)$$

$$\nu_{\bar{q}} \int \frac{\mathrm{d}\mathbf{v}}{4\pi} \langle Q_-^{(1)}(k)\mathbf{v}\cdot\mathbf{A}(k')\rangle = g \int \frac{\mathrm{d}k_1\mathrm{d}k_2}{(2\pi)^8}\delta(k-k_1-k_2)\mathrm{Im}\int \frac{\mathrm{d}\mathbf{v}}{4\pi}\langle[\mathbf{v}\cdot\mathbf{A}(k_2), Q_-^{(2)}(k_1)]\mathbf{v}\cdot\mathbf{A}\rangle, \quad (8)$$

$$\nu_g \int \frac{\mathrm{d}\mathbf{v}}{4\pi} \langle G^{(1)}(k)\mathbf{v}\cdot\mathcal{A}(k')\rangle = g \int \frac{\mathrm{d}k_1\mathrm{d}k_2}{(2\pi)^8}\delta(k-k_1-k_2)\mathrm{Im}\int \frac{\mathrm{d}\mathbf{v}}{4\pi}\langle[\mathbf{v}\cdot\mathcal{A}(k_2), G^{(2)}(k_1)]\mathbf{v}\cdot\mathcal{A}\rangle. \quad (9)$$

One should note that for a baryonless plasma the numbers of quarks and antiquarks are equal to one another and $\nu_q = \nu_{\bar{q}}$ (We will omit variable k_i in any a quanta when the equations are mistaken in next text). Therefore ν_q is expressed as

$$\begin{aligned} \nu_q &= g^2 \int \frac{\mathrm{d}k'\mathrm{d}k_1\mathrm{d}k_2\mathrm{d}k_3}{(2\pi)^{12}}\delta(k-k_1-k_2-k_3) & (10) \\ &\times \frac{\int \frac{\mathrm{d}\mathbf{v}}{4\pi}\mathrm{Im}(\frac{1}{\omega_3+\omega_1-\mathbf{k_3}\cdot\mathbf{v}-\mathbf{k_1}\cdot\mathbf{v}})(\frac{\omega_3}{\omega_3-\mathbf{k_3}\cdot\mathbf{v}} - \frac{\omega_1}{\omega_1-\mathbf{k_1}\cdot\mathbf{v}})f_{aed}f_{bce}\langle\mathbf{v}\cdot\mathbf{A}_a\mathbf{v}\cdot\mathbf{A}_b\rangle\langle\mathbf{v}\cdot\mathbf{A}_c\mathbf{v}\cdot\mathbf{A}_d\rangle}{\int \frac{\mathrm{d}k'\mathrm{d}\mathbf{v}}{4\pi}\frac{\omega}{\omega-\mathbf{k}\cdot\mathbf{v}}\langle 2\mathrm{tr}(\mathbf{v}\cdot\mathbf{A}\mathbf{v}\cdot\mathbf{A})\rangle} & (11) \end{aligned}$$

ν_g is obtained likewise

$$\begin{aligned} \nu_g &= Ng^2 \int \frac{\mathrm{d}k'\mathrm{d}k_1\mathrm{d}k_2\mathrm{d}k_3}{(2\pi)^{12}}\delta(k-k_1-k_2-k_3) & (12) \\ &\times \frac{\int \frac{\mathrm{d}\mathbf{v}}{4\pi}\mathrm{Im}(\frac{1}{\omega_3+\omega_1-\mathbf{k_3}\cdot\mathbf{v}-\mathbf{k_1}\cdot\mathbf{v}})(\frac{\omega_3}{\omega_3-\mathbf{k_3}\cdot\mathbf{v}} - \frac{\omega_1}{\omega_1-\mathbf{k_1}\cdot\mathbf{v}})f_{aed}f_{bce}\langle\mathbf{v}\cdot\mathbf{A}_a\mathbf{v}\cdot\mathbf{A}_b\rangle\langle\mathbf{v}\cdot\mathbf{A}_c\mathbf{v}\cdot\mathbf{A}_d\rangle}{\int \frac{\mathrm{d}k'\mathrm{d}\mathbf{v}}{4\pi}\frac{\omega}{\omega-\mathbf{k}\cdot\mathbf{v}}\langle \mathrm{tr}(\mathbf{v}\cdot\mathcal{A}\mathbf{v}\cdot\mathcal{A})\rangle} & (13) \end{aligned}$$

We can determine the damping rate γ from the following derivation,

$$\begin{aligned} & N_f\nu_q\langle(Q_+^{(1)} - Q^{(1)})\mathbf{v}\cdot\mathbf{A}\rangle + 2\nu_g\tau_a\langle\mathrm{tr}(T_aG^{(1)})\mathbf{v}\cdot\mathbf{A}\rangle \\ = \; & \gamma\left(N_f\langle(Q_+^{(1)} - Q^{(1)})\mathbf{v}\cdot\mathbf{A}\rangle + 2\tau_a\langle\mathrm{tr}(T_aG^{(1)})\mathbf{v}\cdot\mathbf{A}\rangle\right), \end{aligned}$$

and then

$$\gamma = \frac{N_f\nu_q\int \mathrm{d}^3p\partial_p^0Q^R + N\nu_g\int\mathrm{d}^3p\partial_p^0G^R}{N_f\int\mathrm{d}^3p\partial_p^0Q^R + N\int\mathrm{d}^3p\partial_p^0G^R}. \quad (14)$$

Of course, for the case of white noise with same color inherent we have

$$\gamma = g^2N\frac{\int \frac{\mathrm{d}k_2}{(2\pi)^4}\frac{\mathrm{d}\mathbf{v}}{4\pi}\pi\delta[\omega_2-\omega-(\mathbf{k}_2-\mathbf{k})\cdot\mathbf{v}](\frac{\omega_2}{\omega_2-\mathbf{k}_2\cdot\mathbf{v}} - \frac{\omega}{\omega-\mathbf{k}\cdot\mathbf{v}})(\frac{(\mathbf{k}_2\cdot\mathbf{v})^2}{\mathbf{k}_2^2}\langle A_l^2(k_2)\rangle + \frac{(\mathbf{k}_2\times\mathbf{v})^2}{\mathbf{k}_2^2}\langle A_t^2(k_2)\rangle)}{\int\frac{\mathbf{v}}{4\pi}\frac{\omega}{\omega-\mathbf{k}\cdot\mathbf{v}}} \quad (15)$$

where l and t respectively denote the longitudinal and transverse components of the field. Now we discuss the specific results with respect to γ. We know

$$\mathbf{A}\cdot\mathbf{A}(\omega,\mathbf{k}) = A_l^2(\omega,\mathbf{k}) + A_t^2(\omega,\mathbf{k}),$$

$$\langle A^2(\omega,\mathbf{k})\rangle = I(\mathbf{k})\frac{\pi}{\omega^2}[\delta(\omega-\omega(\mathbf{k})) + \delta(\omega+\omega(\mathbf{k}))],$$

where $I(\mathbf{k})$ represents the correlation intensity of collective fields with frequencies $\omega(\mathbf{k})$ and $-\omega(\mathbf{k})$, here we think only the positive frequency mode exists and $I = 4\pi T$ when the plasma is equilibrium. We discuss longwavelenth modes for three cases

(i) The case of longitudinal wave: $A_l^2 = A^2, A_t^2 = 0$, the dispersive relation is $\omega_l(\mathbf{k}) = \omega_p^2 + \frac{3}{5}\mathbf{k}^2$, then we obtain

$$\gamma_l = 0.01g^2T$$

(ii)The case of transverse wave: $A_l^2 = 0, A_t^2 = A^2$, the dispersive relation is $\omega_t(\mathbf{k}) = \omega_p^2 + \frac{6}{5}\mathbf{k}^2$, we have

$$\gamma_t = 0.43g^2T$$

(iii)We can also calculate γ for the mixed case with $A_l^2 = A_t^2$.

$$\gamma_{l+t} = 0.21g^2T$$

In fact, we can calculate γ for more general case if the oscillational direction of the collective modes with respect to wave vector is given. Although we have not treated general case, we see that the damping for a transverse mode is much stronger than the longitudinal one and the damping rate is on the increase as polarized field direction deviation from the direction of wave motion.

One has been applying 'hard thermal loop" method[4] to discuss the damping rate in high temperature plasma. The order do coincides with the one for equilibrium which we study above in the framework of kinetic theory. It seem to imply a obvious connection to classical nonlinear effect from 'hard thermal loop'. Moreover, our study can be extended to out of equilibrium.

The damping do not arise from collisional process though we apply the relaxation-time method. Here we easily analyzes the physical mechanism of the damping from the above calculations. The previous equations contain the field powers from $(\mathbf{v} \cdot \mathbf{A})^2$ to$(\mathbf{v} \cdot \mathbf{A})^4$. It means the existing of Cherenkov process, decay process and scattering process in interactions of plasmons(collective modes) with the particles. It is possible that all these processes cause particles and plasmon(collective) to exchange the energy and momentum each other. However, we know the Cherenkov process cannot be produced due to $\omega > |\mathbf{k}|$ in linear response approximation[5]. In addition, the decay of plasmon has been proved vanish since $\int \frac{\mathrm{d}\mathbf{v}}{4\pi} \mathbf{vvv} f(v) = 0$. So we emphasize the plasmon scattering is the mechanism of collisionless damping, which represents nonlinear interaction.

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